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MATHEMATICAL MODELS FOR FILM SENSITIVITY MEASUREMENTS

Ernest Hammond, Jr.
Department of Physics
Stephen Gewirtz and Osborne Parchment
Department of Mathematics
Morgan State University
Baltimore, MD

and

Gerald R. Baker
Technical Monitor
Laboratory of Astronomy and Solar Physics
Goddard Space Flight Center

ABSTRACT

The quality of the pictorial record developed from photographic material depends on the composition of the material, the procedures used in its exposure and processing as well as the nature of the physical and temporal environment extant during the creation of the record. By holding many of the variables fixed, this paper examines the effect of two environmental parameters, namely temperature and aging, on the characteristic curve of a given film. Polynomial and exponential functions are evaluated as empirical formulas for the characteristic curve, and the sensitivity of derived coefficients to the selected parameters is assessed.

are examined

MATHEMATICAL MODELS FOR FILM SENSITIVITY MEASUREMENTS

I. Data Description and Rationale for Data Analysis

In a variety of applications of scientific photography, there is an unavoidable time lapse between film exposure and film processing. During this time, the exposed film may be subject to varying levels of ambient temperature. Such is the case, for example, when films of experiments are exposed at various times during a space mission and developed at the end of the mission. A similar scenario occurs when films are exposed at distant experimental sites and returned by common carrier to a central location for processing. In such cases the extent to which the photographic record may be affected by the aging and/or the temperature is an important consideration. The data used in this analysis was developed to assess the effect of these physical parameters on one type of common film. A description of the method used follows.

Film of the type selected was exposed for a fixed time using a light sensitivity wedge of thirty gray levels. Following exposure, the film was aged for a selected number of days at a selected temperature and then developed. The density for each gray level in the wedge was measured using a densimeter to produce data similar to that shown in Table I. For this analysis, aging periods were assigned in multiples of three days to a maximum of twenty days and the incubation temperatures were eleven and forty degrees. The development procedure was identical for all the films used in this phase of the project. The actual film used to generate the density readings used in this report was KODAK IIaO.

The density readings accurate to two decimal places, as shown, are dependable for the method as outlined. No attempt is made to correct for differences in storage times between film manufacture and film exposure. However, the supplier maintained manufacturing and shipping practices designed to minimize the effects of this factor [1].

II. Selection, Implementation and Testing of Empirical Models

The earliest researchers in the science of photography realized that a functional relationship exists between photographic densities and the exposures which produce them. F. Hurter and V. Driffield [2] originated the method of plotting density against the common logarithm of exposure to obtain empirically the characteristic curves (Figure I) so central to the theory of sensitometry. Researchers in population biology may also notice a strong similarity to the logistic growth curve.

Despite many attempts to describe this curve globally by a single mathematical formula, no satisfactory functional relationship has so far been commonly accepted as theoretically based and empirically accurate. Consequently, this study uses regression analysis on a selection of curves to represent the experimental data and presents the root mean square error resulting from each choice.

WEDGE NUMBER	DENSITY (15 days, 11°C)	DENSITY (15 days, 40°C)
0	0.22	0.31
1	0.23	0.31
2	0.24	0.32
3	0.25	0.34
4	0.27	0.38
5	0.30	0.41
6	0.35	0.51
7	0.39	0.57
8	0.46	0.66
9	0.53	0.73
10	0.62	0.82
11	0.72	0.91
12	0.78	1.00
13	0.90	1.10
14	1.03	1.19
15	1.13	1.26
16	1.25	1.35
17	1.35	1.43
18	1.47	1.56
19	1.60	1.64
20	1.68	1.69
21	1.75	1.74
22	1.82	1.78
23	1.88	1.82
24	1.93	1.91
25	1.99	1.94
26	2.03	1.97
27	2.07	2.00
28	2.11	2.03
29	2.14	2.04

TABLE I
Density Values for KODAK IIaO Film Aged 15 Days at 11, 40 Degrees Celcius.

A. Polynomial Fit by Regression Analysis

Although characteristic curves exhibit segments which appear to be approximately linear, the graph of density versus the logarithm of admissable exposures is obviously not globally a straight line. Thus an attempt to model these curves by polynomials requires them to be at least quadratic. In fact, the presence of at least one inflexion point suggests that cubics are the minimum needed if a reasonably accurate fit is to be assured. Consequently, this analysis begins with the selection of the most general cubic polynomial as a possible regression curve and proceeds to test higher degree functions until there is no perceptible change in the root mean square error computed.

The regression method for fitting a polynomial

$$P_N(X) = C_0 + C_1 X + C_2 X^2 + \ldots C_N X^N$$

to the data set (X_K, Y_K) $k = 0...N$, involves finding the coefficients C_K so that the expression for the least square error

$$LSE = (Y_K - P_N(X_K))^2$$

or for the root mean square error

$$RMSE = (LSE/N)$$

is minimum. This is a standard procedure in numerical or statistical analysis. The procedure results in a set of M+1 simultaneous, linear equations for the unknown coefficients. Using the Gauss-Jordan method [5] for solving such equations, the unknown coefficients which determine the best fitting polynomial are determined. The algorithms to do this were coded in Pascal and the code may be obtained by writing to the authors. Figure II shows the experimental data together with cubic and quintic regression curves.

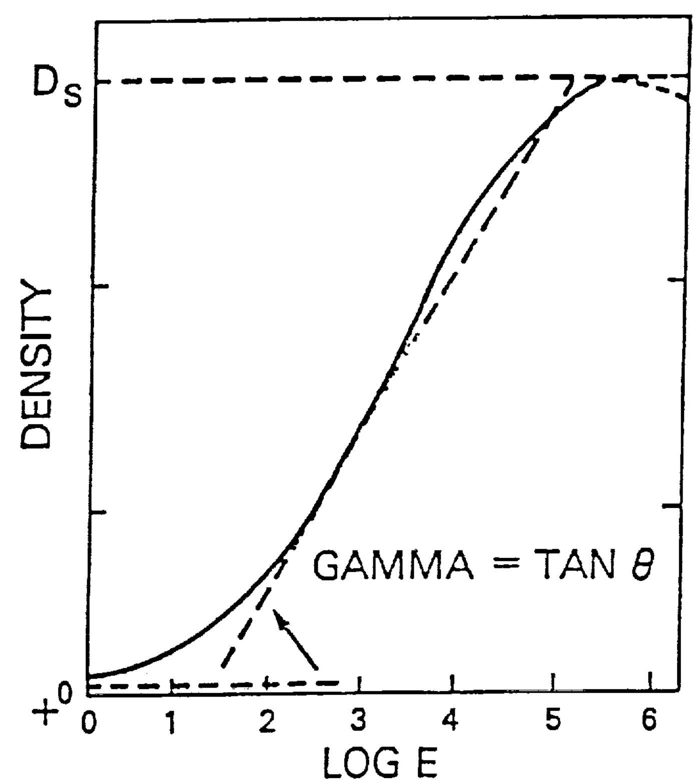

FIGURE I: Typical Characteristic Curve
Film Density vs. Log of Exposure.

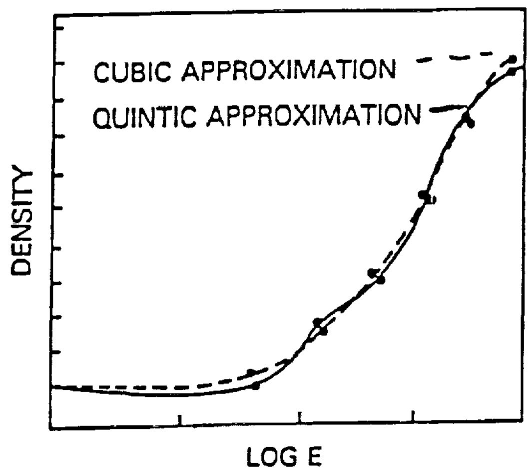

FIGURE II. Two Polynomial Approximation Curves.

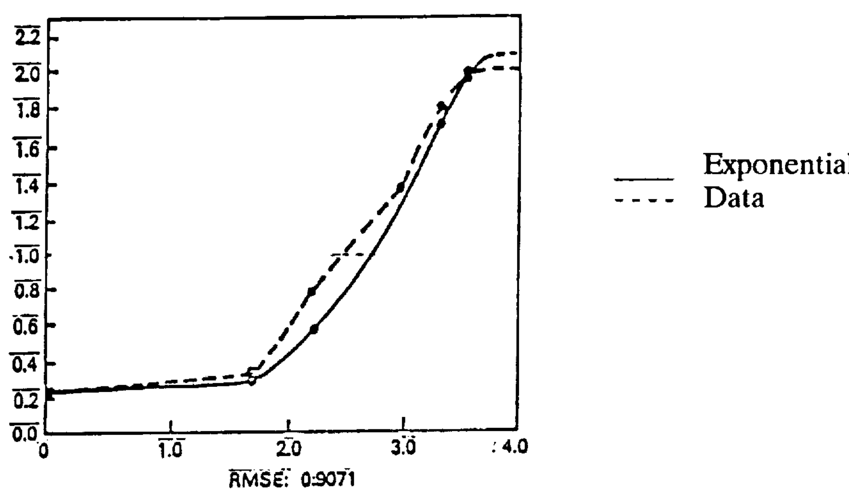

FIGURE III. Exponential Regression Fit to Data.

Film Density Data Approximated by Selected Polynomials

B. Exponential Fit by Regression Analysis

The polynomial curves derived in the preceding section produce a very good fit with the experimental sensiometric data. However, the simplest of the models considered requires the determination of four unknown coefficients. A useful question to consider is the existence of other curves capable of producing an acceptable fit with fewer adjustable constants. The general nature of the characteristic curves suggests that the rate of change of density y with respect to the logarithm of exposure x should have the form

$$* \qquad dy/dx = y(a-by)$$

where a and b are unknown constants. Viewing this as a differential equation for y in terms of x, we could impose the initial condition:

$$y(0) = \text{the fog density of the film.}$$

Solving the resulting initial value problem using methods outlined for example in [6], yields the general solution:

$$** \qquad y = a/[b + (a/y(0) - b)\exp(-ax)].$$

The problem is to determine the constants a, b from the experimental data. It will be helpful to note two relevant properties of the solutions of equation *.

Property I: For large values of x, the values of y approach the ratio a/b.

Property II: Any solution has a flexpoint at the value $y = a/(2b)$ and the rate of change at this flexpoint is $a^2/(4b)$.

The first is obvious from the solution formula **; the second can be derived by the substitution of a/(2b) for y in the right hand side of *. A reasonable assumption is that they approximated the linear portion of the characteristic curve. Using this assumption together with Property I & II gives:

$$a^2/(4b) = \text{slope of linear regression line}$$

$$a/b = \text{maximum density of exposed film}$$

Consequently, the values for the constants a and b are determined by the data. Figure III compares the experimental data with the exponential regression curve derived as described above.

C. Comparision of the Models

Analysis of the approximation data shows a very good agreement of the experimental data with cubic polynomials. Although there is evident improvement resulting from the use of quintic regression curves, the changes are not significant enough to warrant the significant amount of additional computation required. Futhermore, extending mathematical approximations to accuracies beyond the error tolerance of the measuring instruments is not generally a useful exercise.

The results obtained by exponential regression show a considerably higher error when compared to the polynomial regression figures. When tempered by the fact that this method needs the determination of only two arbitrary constants compared to four and six respectively in the polynomial cases considered, the evaluation of the exponential model improves somewhat. The geometric properties of the exponential regression curve suggest better agreement for longer exposure time.

III. Conclusions and Questions for Further Study

If the commitment has been made to model the data by polynomial regression, it is clear from the study that the quintic polynomial regression provides a better fit than the same procedure applied to cubic ones. However, the increase in accuracy is not dramatic enough to justify the additional complexity and computation time required for the higher degree. Cubic splines, although not used in this analysis, are likely to provide an even better fit, but are not global polynomials although they belong locally to this class of functions. Lagrange polynomials are not practical for this model because of the high degree required to fit the data and the consequent increase in computation time.

Choosing the model by exponential regression has some theoretical appeal. However, the errors generated are an order of magnitude higher when compared to the cubic polynomial fit.

For fixed temperature, the coefficients of the respective regression curves used remained stable as functions of aging. On the contrary, for a fixed age there was significant variation in the respective coefficients as functions of temperature. Because the data was available only for three distinct temperatures, this variation is not enough to support a claim of instability of the coefficients as functions of temperature. Additional analysis with more closely spaced temperature readings would be necessary to support such a claim.

IV. References

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